GENERAL

Will a Large Complex System be Stable?

Gardner and Ashby have suggested that large complex systems which are assembled (connected) at random may be expected to be stable up to a certain critical level of connectance, and then, as this increases, to suddenly become unstable. Their conclusions were based on the trend of computer studies of systems with 4, 7 and 10 variables.

Here I complement Gardner and Ashby's work with an analytical investigation of such systems in the limit when the number of variables is large. The sharp transition from stability to instability which was the essential feature of their paper is confirmed, and I go further to see how this critical transition point scales with the number of variables $n$ in the system, and with the average connectance $C$ and interaction magnitude $\alpha$ between the various variables. The object is to clarify the relation between stability and complexity in ecological systems with many interacting species, and some conclusions bearing on this question are drawn from the model.

But, just as in Gardner and Ashby's work, the formal development of the problem is a general one, and thus applies to the wide range of contexts spelled out by these authors.

Specifically, consider a system with $n$ variables (in an ecological application these are the populations of the $n$ interacting species) which in general may obey some quite nonlinear set of first-order differential equations. The stability of the possible equilibrium or time-independent configurations of such a system may be studied by Taylor-expanding in the neighbourhood of the equilibrium point, so that the stability of the possible equilibrium is characterized by the equation

$$\frac{dx}{dt} = Ax$$

Here in an ecological context $x$ is the $n \times 1$ column vector of the disturbed populations $x_j$, and the $n \times n$ interaction matrix $A$ has elements $a_{jk}$ which characterize the effect of species $k$ on species $j$ near equilibrium. A diagram of the trophic web immediately determines which $a_{jk}$ are zero (no web link), and the type of interaction determines the sign and magnitude of $a_{jk}$.

Following Gardner and Ashby, suppose that each of the $n$ species would by itself have a density dependent or otherwise stabilized form, so that if disturbed from equilibrium it would return with some characteristic damping time. To set a time-scale, these damping times are all chosen to be unity: $a_{jj} = -1$.

Next the interactions are "switched on", and it is assumed that each such interaction element is equally likely to be positive or negative, having an absolute magnitude chosen