A Note on the Cobb-Douglas Function

By

Herbert A. Simon and Ferdinand K. Levy

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In economic models incorporating production functions, the form of the function most commonly used is that proposed by Cobb and Douglas [3]:

$$P = bL^{k} C^{1-k} \tag{1}$$

It is a well known implication of this function that labor's fraction of total product is equal to the exponent, k. For the marginal productivity of labor, $\frac{\partial P}{\partial L}$, equal to the competitive wage, is simply:

$$\frac{\partial P}{\partial L} = k \frac{P}{L} \tag{2}$$

which multiplied by L yields kP as labor's share.

Thus, it has been thought that an empirical test of the validity of the Cobb-Douglas assumptions could be made by comparing the value of k obtained by fitting (1) to empirical data on (P, L, C) with the value of k obtained by a direct measure of labor's share of income. (We shall call the latter the " actual " k.) Generally, the two values of k obtained in these ways correspond very closely [2, 3]. In fact, Bronfenbrenner and Douglas [2] point out the consistency of the value of k in fitting the function to both time series data ($k = .75$) and cross section data of ninety firms ($k = .74$), both of which are in general agreement with the actual k of the period studied. Further, when Cobb-Douglas functions were fitted to each of the seven industry groups that comprised the ninety firms, the range of differences between actual and computed labor share ran from (—.37 to —.01).

In this note, we wish to argue that the agreement between the two values of k can be interpreted in quite a different way, and thus does not provide convincing support for the Cobb-Douglas assumptions.[1] Suppose the underlying " true " production function were not Cobb-Douglas, but rather the simple linear function:

$$P = aL + dC \tag{3}$$

Then labor's fraction of income is simply:

$$k' = \frac{aL}{P} \tag{4}$$

Note that this is not too far-fetched an assumption for the form of the production function, especially for cross section data. The constants a and d can be regarded as the average wage and yield on capital respectively when fitted to either intra or inter industry data. (The latter, of course, would yield higher standard errors for the coefficients, but still should provide a good fit to the data.)

[1] A similar point is made by Phelps-Brown [6, pp. 557, *et seq.*], but since he states his argument only briefly, it is not clear whether his line of reasoning is the same as that developed rigorously here.

If we were now, mistakenly, to approximate a fitted (3) with a Cobb-Douglas function, the approximation should be close to (3) at the average values of (P, L, C), say $(\bar{P}, \bar{L}, \bar{C})$. If we use as our approximation a Cobb-Douglas function tangent to (3) at $(\bar{P}, \bar{L}, \bar{C})$, then it is easy to show that the k in the Cobb-Douglas function is equal to the k' given by (4). For if we expand both (1) and (3) in a Taylor series about $(\bar{P}, \bar{L}, \bar{C})$ and equate the coefficients of linear terms, (1) becomes

$$P = \bar{P} + k\frac{\bar{P}}{\bar{L}}(L - \bar{L}) + (1 - k)\frac{\bar{P}}{\bar{C}}(C - \bar{C}) \tag{5}$$

and (3) becomes

$$P = \bar{P} + a(L - \bar{L}) + d(C - \bar{C}) \tag{6}$$

We then obtain:

$$a = k\frac{\bar{P}}{\bar{L}} \tag{7}$$

and

$$d = (1 - k)\frac{\bar{P}}{\bar{C}} \tag{8}$$

Solving for k yields:

$$k = a\frac{\bar{L}}{\bar{P}} \tag{9}$$

or $k = k'$ which was to be shown.

Thus, the existence of a fitted Cobb-Douglas function with a value of k in agreement with the actual k does *not* imply that the underlying production function is truly Cobb-Douglas. In fact, we expect this agreement when the true function is given by (3).

Pittsburgh.

HERBERT A. SIMON.
FERDINAND K. LEVY.

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